TOPIC III

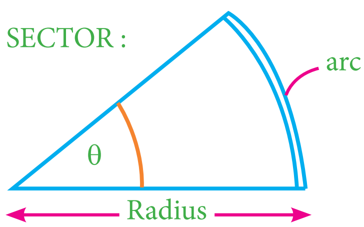
FULL NOTES ⇒ CIRCULAR FUNCTIONS & TRIGONOMETRY

3.1 - CIRCLE BASICS

➔ **FUNCTION** ⇒ Alternative way of measuring degrees.

↳ π radians = 180° [2π = 360° , $\pi/2$ = 90°]

➔ **OTHER RULES** ⇒



ARC LENGTH : $L = \theta r$ (in radians)

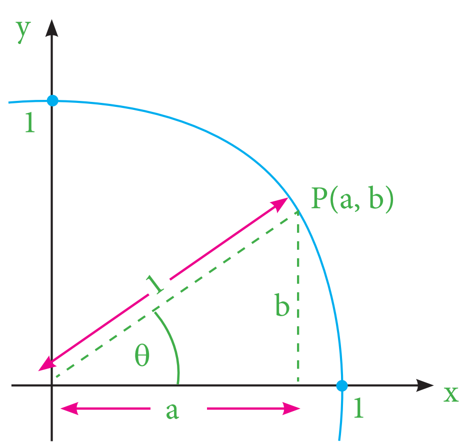
AREA of SECTOR : $A = 1/2\ \theta r^2$

Both in formula booklet

3.2 - UNIT CIRCLE

➔ **UNIT CIRCLE** ⇒ Defined as a circle with its center at the origin and with a radius of 1.

➔ **REALITIONSHIP WITH SIN / COS** ⇒ Looking at the diagram below, we can analyse the right - angled triangle created by the point P(a, b).



Using SOHCAHOA : $\sin\theta = \frac{O}{H} = \frac{b}{1}$

$\cos\theta = \frac{A}{H} = \frac{q}{1}$

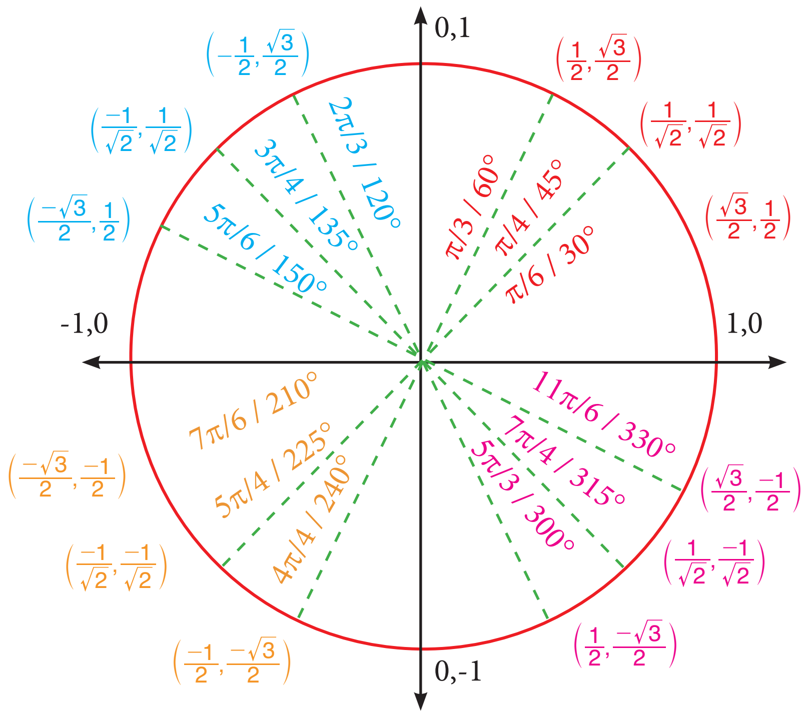
SO : $a = \cos\theta$
$b = \sin\theta$

or P(a, b) ⇒ P(cosθ , sinθ)

→ **SPECIFIC VALVES** ⇒ By learning the coordinates on the unit circle at given angles, you can learn important valves of $\sin\theta$ and $\cos\theta$

The diagram to the right shows specific valves with multiples of $\pi/4$ & $\pi/6$

As a full circle is 2π , these valves repeat evr 2π



3.3 INDENTTITIES

→ **TRIGONOMETRIC**

IDENTITY ⇒ Looking back to the trianle in the unit circle :

$\tan\theta = \frac{opp}{adj}$ ⇒ $\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}}$

EG - 1 ⇒ Calculate $\frac{\sin\pi}{\cos\pi}$: $\frac{\sin\pi}{\cos\pi} = \tan\pi = 0$

EG - 2 ⇒ Simplify 3sinx + 2cosx.tanx :

$3\sin x + 2\cos x \left(\frac{\sin x}{\cos x}\right) = 3\sin x + 2\sin x = 5\sin x$

➔ PYTHAGDREAN

IDENTITY $\Rightarrow$ Using the pythagorean theorem on triangle from the unit circle, we get.

$a^2 + b^2 = 1^2 \Rightarrow$	$\cos^2\theta + \sin^2\theta = 1$

EG - 1 $\Rightarrow$ Simplify $\cos^2\theta \sin^2\theta + \sin^3\theta$:

As $\cos^2\theta \; \sin\theta + \sin^3\theta = \sin \; (\cos^2\theta + \sin^2\theta)$

We get $\sin\theta(1) = \sin\theta$

EG - 2 $\Rightarrow$ Simplify $3\sin^2\theta + 3\cos^2\theta$: $3(\sin^2\theta + \cos^2\theta) = 3(1) = 3$

➔ DOUBLE ANGLE

FORMLULAE $\Rightarrow$ We also have formulae for $\sin 2\theta$ & $\cos 2\theta$:

$$\sin 2\theta = 2\sin\theta \cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

OR $= 1 - \sin^2\theta$

OR $= 2\cos^2\theta - 1$

EG - 1 $\Rightarrow$ If $\sin\theta = \frac{4}{5}$ & $\cos\theta = \frac{3}{5}$

a) FIND $\sin 2\theta$: $\sin 2\theta = 2\sin\theta \cos\theta$

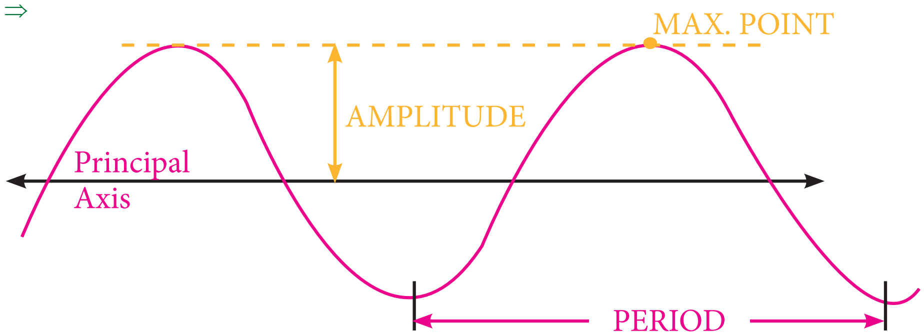
$$= 2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right) = \frac{24}{5}$$

b) FIND $\cos 2\theta$: $\cos 2\theta = 1 - 2\sin^2\theta$

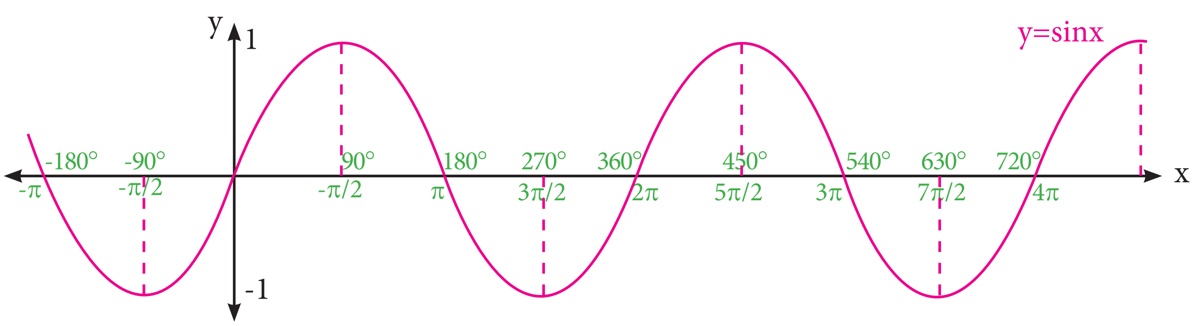
$$= 1 - 2\left(\frac{4}{5}\right)^2 = 1 - \frac{32}{25} = -\frac{7}{5}$$

3.4 TRIG FUNCTION FEATURES

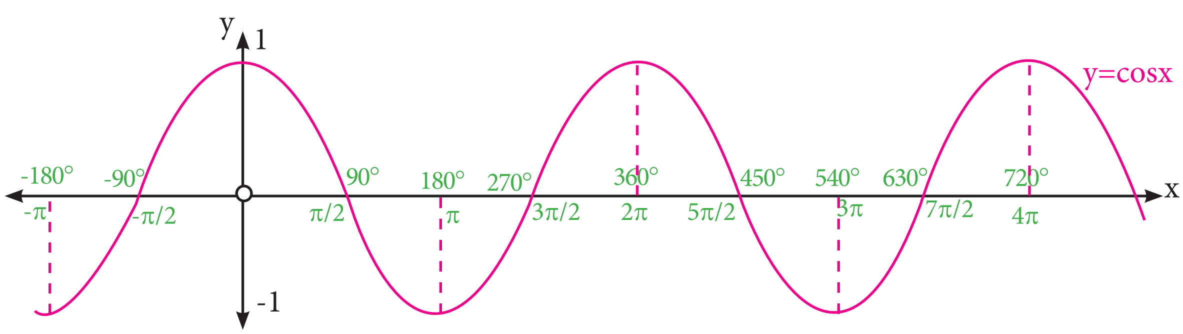
➔ TERMINOLOGY ⇒



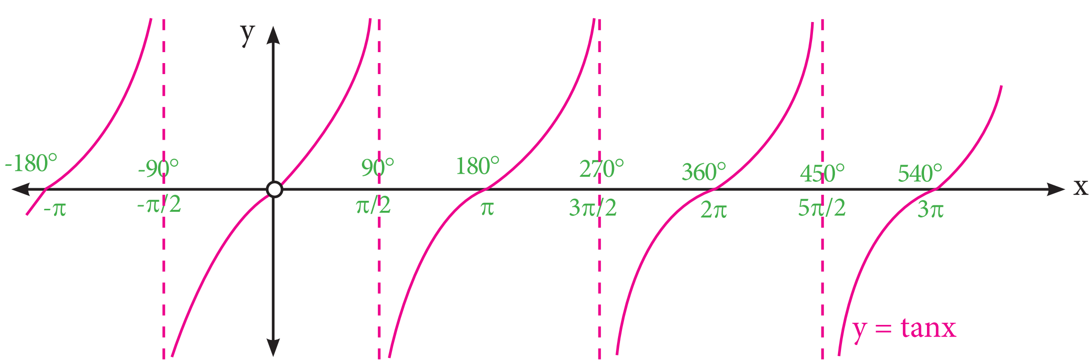
➔ SINE CURVE ⇒



➔ COSINE CURVE ⇒



➔ TAN ⇒



➔ TRANSFORMATIONS ⇒

(Same for cosine) $y = a \sin(b(x - c)) + d$

⇒ Principal axis at $y = d$

⇒ Period is $\frac{2\pi}{b}$

⇒ Amplitude is $|a|$

⇒ Horiz. translation by c

$y = a \tan(b(x - c)) + d$

⇒ Principal axis at $y = d$

⇒ Period is $\frac{\pi}{b}$

⇒ Amplitude undefined

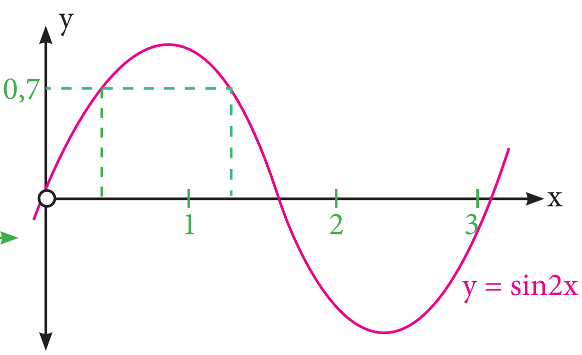
⇒ Horiz. translation by c

3.5 SOLVING TRIG EQUATION

➔ METHOD 1 ⇒ Using Graps

E.G - 1 ⇒ Solve $\sin(2x) = 0.7$ for $0 \le x \le 2$:

Use graph we see $x \approx 0.4$ & 1.2



➔ METHOD 2 ⇒ Using tecnology

E.G - 2 ⇒ Solve $\sin(2x) = 0.7$ for $0 \le x \le 2$:

There are a couple of ways with GDC :	
a)	Enter in 2 lines $Y_1 = \sin 2x$ & $Y_2 = 0.7$ ↳ Find intersection $x = 0,3877$ & 1.1731
b)	Rearrange to get $x = \frac{\sin^{-1}(0,7)}{2}$ and use GDC to calculate this valve.

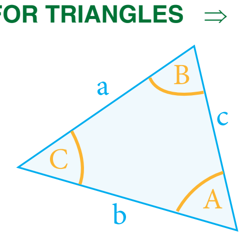
➔ METHOD 3 ⇒ Using algebraic methods

E.G - 3 ⇒ Solve $\sin(2x) = -\frac{1}{2}$ for $0 \le x \le 2\pi$:

We will need to observe the unit circle for when the y - coordinate is -1/2.
We must make on adjustment as we are looking 2x. so the range of angles doubles, to o 4π. From the unit circle,
$\sin\theta = -\frac{1}{2}$ at $\frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}$. We must halve these to reach our answers for
$x : \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

3.6 TRIANGLE TRIG

➔ FOR TRIANGLES ⇒



AREA RULE : $\text{Area} = \frac{1}{2} ab\sin C$

SINE RULE : $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

COSINE RULE : $a^2 = b^2 + c^2 - 2bc(\cos A)$ OR $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$