FULL NOTES ⇒ FUNCTIOS

2.1 - CONCEPTS

➔ **FUNCTION** ⇒ When a relaktion is given as an equation, if each x valve gives you only 1 y - valve, it is a FUNCTION

➔ **NOTATION** ⇒ These are three ways of writing the same function :

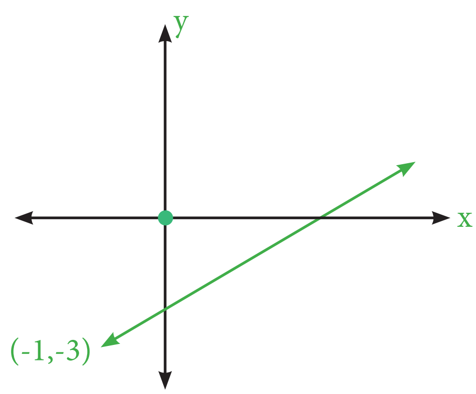
$f : x \to 2x + 3$ // $f(x) = 2x + 3$ // $y = 2x + 3$

➔ **DOMAIN**

➔ **RANGE** ⇒ **Domain :** Set of x -valves you can enter into be function.

Range : Set of y - valves you can enter into the function.

EG - 1 ⇒



Domain : $\{x \mid x \geq -1\}$

Range : $\{y \mid y \geq -3\}$

EG - 2 ⇒ $y = \frac{1}{x-5}$

Domain : $\{x \mid x \neq 5\}$ (can't divide by zero)

Range : $\{y \mid y \neq 0\}$ (this fraction can't equal zero)

➔ **COMPOSITE** ⇒ If we take two functions, f(x) & g(x), a composite function will convert x into $f(g(x))$

We represent this with $f(g(x)) = (fog)(x)$

EG - 1 ⇒ $f(x) = 2x + 1$, $g(x) = 3 - 4x$: $(fog)(x) = 2(3 - 4x) + 1 = 6 - 8x + 1$

$= 7 - 8x$

EG - 2 ⇒ $f(x) = 6x - 5$, $g(x) = x^2 + x$: , find (gof)(1) :

$(gof)(x) = (6x - 5)^2 + (6x - 5)$ ⇒ $(gof)(1) = (1)^2 + (1) = 2$

➔ INVERSE ⇒ You can find the inverse of a function by switching x & y, then making y the subject again.

EG - 1 ⇒ If $f(x) = 2x + 3$, find $f^{-1}(x)$: $y = 2x + 3$ $x = 2y + 3$, $x - 3 = 2y$

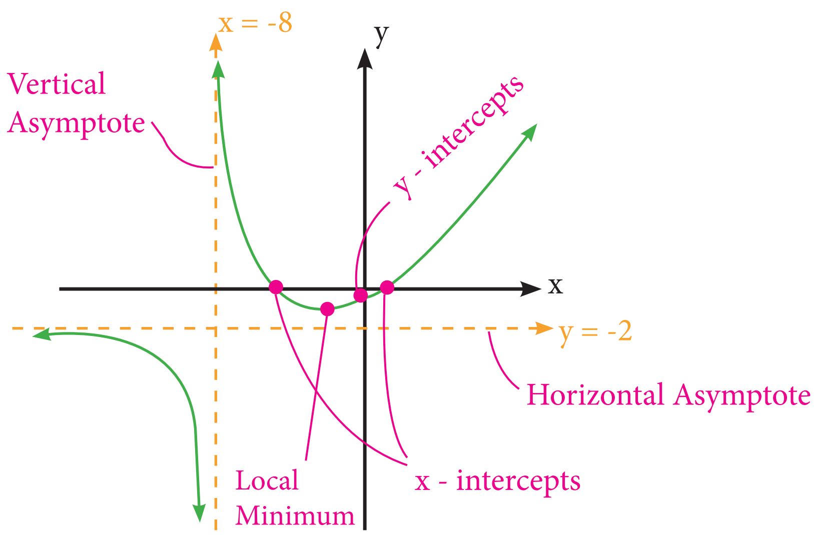
$$y = \frac{x-3}{2} \Rightarrow f^{-1}(x) = \frac{x-3}{2}$$

NOTE : * You can find it graphically by reflecting in $y = x$

* You can test an inverse by using the rule : $(fof^{-1})(x) = x$

2.2 - GRAPH FEATURES

➔ IDENTIFY FEATURES ⇒



Domain : $\{x \mid x \neq -8\}$

Range : $\{y \mid y \neq -2\}$

➔ INTERSECTION ⇒ You can find where the graphs f(x) 8.g(x) meet by setting f(x) = g(x), the solving.

EG - 1 ⇒ Find intersections of $f(x) = x^2 + 4$ & $g(x) = -5x$

Set : $f(x) = g(x)$: $x^2 + 4 = 5x$, $x^2 + 4 + 5x : 0$

Factoriese : $x^2 + 5x + 4 = 0$: $(x + 4)(x + 1) = 0$, $x = -1$ or -4

f(x) & g(x) meet at $x = -1$ and -4

2.3 TRANSFORMATION

➔ **RULES :** The following rules are **NOT** in the formula booklet.

TRANSLATION :	y = f(x) + b ⇒	Shift up by b units
	y = f(x - a) ⇒	Shift right by b units
STRETCHES :	y = p f(x) ⇒	Stretch vertically by scale factor p
	y = f (q x) ⇒	Stretch horizontally by scale factor 1/2
REFLECTIONS :	y = -f(x) ⇒	Reflect in the x - axis
	y = f(-x) ⇒	Reflect in the y - axis

EG - 1 ⇒ Decribe transformation of $y = x^2$ to $y = (x - 8)^2$

Translation to the right by 8 units

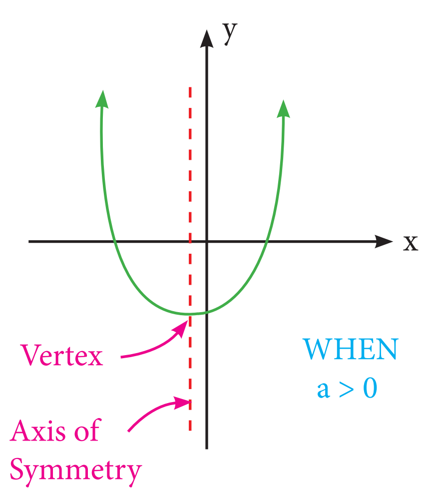
EG - 2 ⇒ Write a function to stretch $y = \sqrt{x}$ vertically by s.f.2, and reflect in tı y - axis

$y = 2\sqrt{x}$

2.4 QUADRATICS

➔ **GENERAL FORM :** A quadratic is usually written as : $f(x) = ax^2 + bx + c$

GRAPH ⇒



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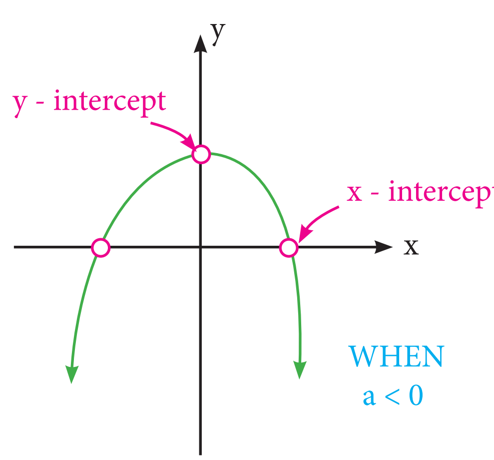


➔ **FINDING FEATURES** $\Rightarrow$ **y - intercept :** found at (0,c)

Axis of symm : found at $x = \frac{-b}{2a}$ This is alsa the x - coord of te vertex

x - intercept : set y = 0 and solve with : factorising guad . form

$$w / f(x) = ax^2 + bx + c$$

EG - 1 $\Rightarrow$ Find intersections of $y = 3x^2 - 11x - 4$:

y - int : (0, -4) / Axis of symm : $x = \frac{-b}{2a} = \frac{11}{6}$

x - int : $3x^2 - 11x - 4 = 0$, $(3x + 1)(x - 4) = 0$, at $x = 4$ or $-\frac{1}{3}$

NOTE : Quad, formula is $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and as we can't have square roots of a negative : If b2 - 4ac < 0, there are no roots

➔ **FEATURES** $\Rightarrow$ **x - intercept :** at (p,0) & (q, 0)

vertex : at $x = \frac{p+q}{2}$

$$w / f(x) = a(x - p)(x - q)$$

➔ **FEATURES** $\Rightarrow$ **vertex :** at (h, k)

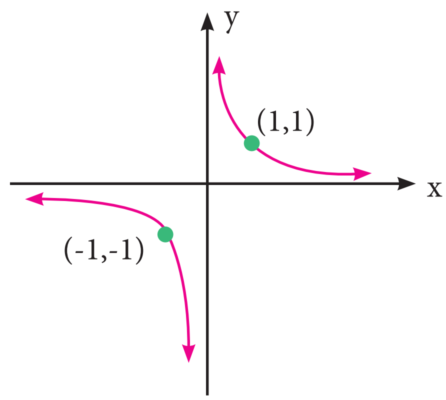
$$w / f(x) = a(x - p)(x - q)$$

NOTE : You will need to use these formulae for finding the vertex to help with real - wold optimisation problems.

2.5 RATIONALS

➔ **RECIPROCAL** ⇒ is a function in the form $y = \frac{k}{x}$, and simplet form of this is

$y = \frac{1}{x}$



⇒ Symmetrical through $y = \frac{1}{x}$

↳ So it is its own inverse

⇒ Asymptotes on each axis

⇒ Passes through (1,1) & (-1, -1)

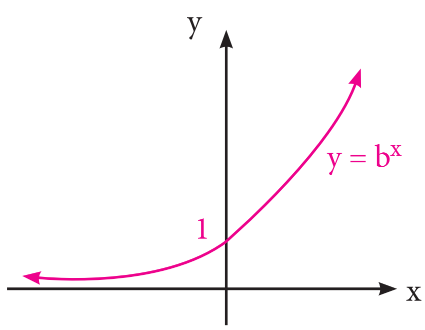
$y = \frac{ax+b}{cx+a}$ ⇒ It shifts this same shape and strectches it. It has the following features :

Vertical asymp. at $x = \frac{-d}{c}$ // Horizantal asymp. at $y = \frac{a}{c}$

EG - 1 ⇒ $y = \frac{2x-5}{2x-8}$ has vert asym. at $x = \frac{8}{2} = 4$ // H. A. at $y = \frac{2}{2} = 1$

2.6 EXP./LOG GRAPHS

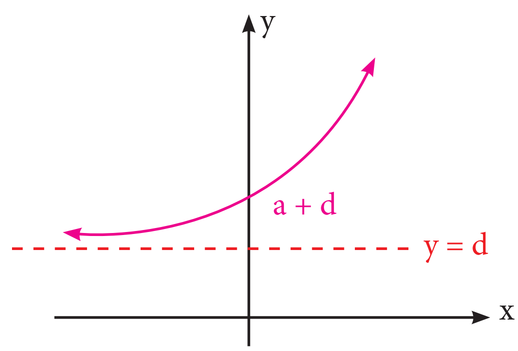
➔ **SIMPLE EXPONENT** ⇒ $y = b^x$



⇒ HORIZ. ASYM. AT X - AXIS

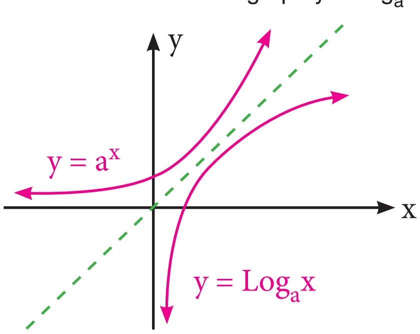
⇒ Y - INT. AT (0,1)

GENERAL EXPONENT FUNCTION ⇒ $y = a(b^x) + d$



⇒ Y - INT AT (0, a + d) ⇒ H. A. at y = d

➔ **LOG GRAPHS** ⇒ The graph $y = \mathrm{Log}_a x$ is the inverse of $y = a^x$



⇒ It is reflected in y = x

$y = \mathrm{Log}_a x$

⇒ Vert . Asym. at y - axis

⇒ X - INT at (1, 0)